Blind Receiver for Space-Time Differentially-Encoded CDMA Systems on Multipath Fading Channels

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Abstract—In this paper, we consider a MIMO multiuser system in a multipath fading channel. We design a blind receiver for a differential space-time coded modulation. A linear estimator projects the received signal onto the same subspace of the MMSE filter. The system works because of the differential encoding of the information matrices. Numerical results show that the blind receiver has the same performance of the unknown MMSE filter.

I. INTRODUCTION

In this paper, we consider a MIMO multiuser system in a multipath fading channel. We design a blind receiver for a differential space-time coded modulation. A linear estimator projects the received signal onto the same subspace of the MMSE filter. The system works because of the differential encoding of the information matrices. Numerical results show that the blind receiver has the same performance of the unknown MMSE filter.

In this context we propose a turbo TCM DUST scheme for a code-division multiple access (CDMA) multipath fading scenario. At the transmitter side, the information bits of each of the $K$ users are first encoded by a standard convolutional encoder, then interleaved and subsequently fed into a unitary space-time trellis modulator. After channel interleaving, the matrices are differentially encoded, spread with the user's short spreading sequence and transmitted on the channel. Thanks to channel interleaving, the fading coefficients affecting a given code sequence can be considered as independent [15].

At the receiver side, sufficient statistics are obtained for each user by sampling the received signal. Then, single-user decoding is performed. A blindly computed linear filter, akin to the MMSE estimator, is then applied to estimate the transmitted symbols. After metric computation channel deinterleaving is performed. For the user of interest, the deinterleaved metrics feed a soft-input soft-output (SISO) trellis demodulator, to start an iteration of the Turbo-like decoder.

The design of the linear estimator is based on a previous paper [4]. In that paper, the receiver design concerns a system with a single transmit antenna per user. The new linear estimator and the standard linear MMSE estimator lie in the same subspace. Similar subspace-based algorithms in different settings can be found in the literature (see [11] and references therein). In our case, the system works because of the differential encoding of the information matrices.

In section 2 we introduce the system model in matrix form. In section 3, the transmitter for our scheme is described in some detail. In section 4, the receiver is designed, with particular attention to the description of the algorithm for finding the estimator. In Section 5, simulation results are discussed. Finally, in Section 6, we draw some conclusions. Throughout the paper, $I$, $H$, $\otimes$, $\text{tr}$, $\text{Re}$ denote transpose, transpose conjugate, Kronecker product, trace and real part, respectively. Matrices and vectors are typed with upper and lower case boldface. $I_n$ is the $n \times n$ identity matrix. $0_{m \times n}$ is the all-zero $m \times n$ matrix.

II. SYSTEM MODEL

Let us consider $K$ synchronous users transmitting simultaneously on the same channel with $t$ antennas. Each user employs a short spreading sequence with spreading factor $N$. We model the received signal as the superposition of $L$ replicas through $L$ paths, each undergoing independent fading. If $r$ is the number of receive antennas, the $l$-th path, $l=0, \ldots, L-1$, of user $k$ can be described by an $r \times t$ matrix $G_k(l)$, whose entries are independent identical distributed (i.i.d.) circular complex Gaussian rv's with variance $a_{kl}^2$, where $a_{kl}$ is the
The convolution of the spreading sequence \(c_k(p), p = 0, \ldots, N - 1\) with the channel matrices will be represented by \(Q \times r\) matrices, where \(Q = N + L - 1\), the \(n\)-th of which, \(n = 0, \ldots, Q - 1\), being:

\[
H_k(n) = \sum_{p=0}^{N-1} c_k(p)G_k(n - p),
\]

or, in matrix form, as shown in (2). The expression enclosed in round brackets in the right-hand side of (2), which will be called \(C_k\), is the \(rQ \times rL\) Kronecker product of the \(Q \times L\) matrix \(C_k\) and the order-r identity matrix, the rightmost matrix is \(G_k\) and the \(rQ \times t\) matrix in the left-hand side will be denoted \(H_k\).

Denoting with \(M = 2R + 1\) the number of transmitted symbols, the received discrete-time baseband signal at the receive antennas at the \(n\)-th time interval is given by

\[
x(n) = \sum_{k=1}^{K} \sqrt{\frac{\rho_k}{t}} \sum_{m=-R}^{R} H_k(n - mN)s_k(m) + z(n),
\]

where:

- \(s_k(m)\) is a \(t \times 1\) vector, collecting the symbols transmitted by user \(k\) from all transmit antennas at time \(m\).
- \(\rho_k\) is the \(k\)-th user’s signal-to-noise power ratio (SNR) at each receive antenna.
- \(z(n)\) is an \(r \times 1\) vector of circular complex unit-variance Gaussian noise samples.

We consider \(P = P_1 + P_2 + 1\) consecutive symbol intervals at the receiver side, where \(P_1 = NR\) and \(P_2 = N(R + 1) + L - 1\), in which the channel is supposed to remain constant. With this hypothesis the received vector can be written more compactly as in (4), where the \(Pr \times 1\) vector in the left-hand side is \(\bar{x}(n)\) and the \(Pr \times 1\) noise vector is \(\bar{z}(n)\). We assume \(rP > KMt\), so that the system is “tall”.

III. TRANSMITTER

The \(k\)-th user encodes his information bit stream \(d_k\) with a convolutional encoder, obtaining the coded bit stream \(b_k\). After interleaving, the interleaved coded bits \(\bar{b}_k\) enter the space-time trellis encoder, whose output is a unitary \(t \times t\) space-time matrix stream \(\{W_k(q)\}\). These matrices are row-by-column interleaved and the output \(\{\tilde{W}_k(q)\}\) is differentially encoded:

\[
X_k(q) = X_k(q - 1)\tilde{W}_k(q)
\]

with \(X_k(0) = I_r\).

The matrix interleaver size is suited to the coherence time of the channel, so that matrices belonging to the same codeword experience independent fading.

Fig. 1 shows the transmitter side for one user in the system. The design of the inner space-time trellis code, whose description follows, is the same as in [1].

A. Unitary Matrices and Set-Partitioning

The space-time encoder consists of a trellis code whose outputs are unitary matrices. As a possible choice, these matrices can be \(2 \times 2\) Alamouti-like defined by two QPSK symbols, resulting in a set of 16 unitary matrices:

\[
W(x, y) = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{-j\frac{\pi}{4}x} & e^{j\frac{\pi}{4}y} \\ -e^{-j\frac{\pi}{4}y} & e^{j\frac{\pi}{4}x} \end{bmatrix},
\]

for \(x, y \in \{0, 1, 2, 3\}\).

The correspondence between matrices and transitions is decided using the set-partitioning method [3], [10]. In Fig.3 the trellis structure is shown, where the matrices \(W(x, y)\) are labelled by \(4x + y\).

In Fig. 4, the two partition steps are represented. The splits have been made according to the maximization of the intra-set distance, where we used as a distance between matrices \(A\) and \(B\):

\[
dist(A, B) = |\det(A - B)|
\]
These generalizations are not treated here. An analogous scheme for a different number of transmit antennas can be devised by designing other unitary matrices. Among these, we remember the diagonal matrices [15], the direct product of Alamouti-like matrices and the Cayley codes [7]. These generalizations are not treated here.

IV. RECEIVER

At the receiver side, the symbol \( s_1(n) \) for user 1, the user-of-interest, is estimated through linear subspace MMSE filtering. The filter coefficients are blindly obtained, by assuming that no channel state information (CSI) is available. Only the spreading sequence for user 1 is needed to be known.

The estimate of symbol \( s_1(n) \) will be given by:

\[
\hat{s}_1(n) = M_1^H \tilde{x}(n),
\]

(8)

where \( M_1 \) is the \( rP \times t \) matrix of the linear estimator.

A. The blind estimator

If the channel is known, the MMSE estimator satisfies the Wiener equation

\[
R_\tilde{x}M_1^{\text{MMSE}} = R_{\tilde{x}s_1},
\]

(9)

being

\[
R_\tilde{x} = E \{ \tilde{x}(n) \tilde{x}^H(n) \}
\]

(10)

and

\[
R_{\tilde{x}s_1} = E \{ \tilde{x}(n)s_1^H(n) \}.
\]

(11)

The expected values can be computed from (4). If the statistics of the transmitted symbols satisfy

\[
E \{ s_j(n)s_k^H(m) \} = \frac{1}{L} \delta_{mn} \delta_{jk} I_l,
\]

(12)

the right-hand side of (9) is given by:

\[
R_{\tilde{x}s_1} = \begin{bmatrix}
0_{rNR,t} & \tilde{H}_1 & 0_{rNR,rL} \\
\tilde{H}_1 & 0_{rNR} & \tilde{C}_1 \\
0_{rNR,t} & \tilde{C}_1 & 0_{rNR,rL}
\end{bmatrix} C_1,
\]

(13)

where the \( rP \times t \) matrix in the middle will be referred to as \( \tilde{H}_1 \) and the \( rP \times rL \) matrix in the right-hand side of the above equation will be denoted \( \tilde{C}_1 \).

Then, the columns of \( R_\tilde{x}M_1^{\text{MMSE}} \) belong to the space spanned by the columns of \( \tilde{C}_1 \).

Also notice that the inverse of \( R_\tilde{x} \) has the same left singular vectors of the correlation matrix of the received signal without noise, due to the whiteness of noise. As a consequence, thanks to (9), the columns of \( M_1^{\text{MMSE}} \) lie in the space generated by the columns of the users’ channel matrices in (4).

In our algorithm, we blindly estimate the subspace in which the MMSE estimator lies, by exploiting the two properties cited above:

1) The MMSE estimation lies in the subspace generated by the users’ channel matrices, and
2) \( R_\tilde{x}M_1^{\text{MMSE}} \) lies in the subspace generated by \( \tilde{C}_1 \).

The matrix \( M_1 \) satisfying both requirements is found as a full-rank solution to the following minimization problem:

\[
\min_M \operatorname{tr} \left\{ M^H \left( E_z E_z^H + R_\tilde{x} \tilde{U}_1 \tilde{U}_1^H R_\tilde{x} \right) M \right\},
\]

(14)

where \( E_z \) is a basis of the subspace of the received signal with noise components only and

\[
\tilde{U}_1 = \begin{bmatrix}
I_{rNR} & 0_{rNR,rQ-rL} & 0_{rNR,rNR} \\
0_{rQ,rNR} & \tilde{C}_1^\perp & 0_{rQ,rNR} \\
0_{rNR,rNR} & 0_{rNR,rQ-rL} & I_{rNR}
\end{bmatrix},
\]

(15)

\( \tilde{C}_1^\perp \) being the orthogonal complement of \( \tilde{C}_1 \).

In practice, \( R_\tilde{x} \) can be estimated from the samples of the received signal, while \( E_z \) is obtained from the singular value decomposition, by taking the \( Pr - KMt \) singular vectors associated to the lowest singular values. Moreover, \( \tilde{U}_1 \) can be computed once and for all at the beginning, by exploiting the knowledge of user 1’s spreading sequence \( c_1 \) (see (15)).

Notice that Conditions 1) and 2) are met by any matrix lying in the subspace generated by the columns of \( M_1^{\text{MMSE}} \). Thus, in general \( M_1 \) is not the MMSE solution. Based on this consideration, to further restrict the solution set, we impose that \( M_1^H M_1 = I_r \), and that \( M_1 \) diagonalizes \( R_\tilde{x} \). These constraints are generally sufficient to grant that the solution is unique. This can be useful if the channel estimates are continuously updated.

When the channel matrices for the other users’ symbols and for user 1’s interfering symbols (see (4)) do not lie in the space generated by the columns of \( \tilde{H}_1 \), the algorithm, based on the two conditions 1) and 2), leads to a subspace with dimension \( t \) (see [4] for the analysis in the \( t = 1 \) case).
relationship between the true MMSE solution and the result of the minimization problem in (14) can then be written as:

$$M_1 = M_1^{(\text{MMSE})} \Theta,$$

where Θ is a $t \times t$ unknown invertible matrix, whose value only depends on the unknown channel matrix $G_1$ in (13), and, thus, stays constant within the channel coherence time.

After filtering as in (8), the metrics are computed and, after channel deinterleaving, fed into the turbo decoder. The turbo decoder consists of the space-time SISO decoder and the outer SISO decoder, which exchange soft information on the transmitted bits along the iterative process.

B. The space-time turbo decoder

Before channel interleaving, the matrices $Y_1(q)$ are built up from the estimated symbols:

$$Y_1(q) = [\hat{s}_1(q), \ldots, \hat{s}_1(qt + t - 1)].$$

The space-time SISO (ST-SISO) decoder has two inputs:

1) the log-likelihood ratios (LLRs) from the channel, defined as

$$\text{LLR}_j^q(q) = \ln \frac{\Pr\{Y_1(q) | W_j, Y_1(q - 1)\}}{\Pr\{Y_1(q) | W_0, Y_1(q - 1)\}},$$

$$j = 1, \ldots, \Gamma - 1$$

where Γ is the number of unitary matrices and $W_0$ is any of them. The explicit expression of the LLRs will be given in Sect. IV-C.

2) the a priori (AP) LLRs on the input bits

$$\text{AP}_l^q = \ln \frac{\Pr\{b_{1,l} = 1\}}{\Pr\{b_{1,l} = 0\}},$$

which are computed in the previous iteration by the outer SISO decoder. In the first iteration, where no a priori knowledge is available, the APs are all set to zero.

The output of the ST-SISO is the extrinsic information on the transmitted bits

$$\text{ExInf}_l^q = \ln \frac{\Pr\{Y_1(q) | b_{1,l} = 1, Y_1(q - 1)\}}{\Pr\{Y_1(q) | b_{1,l} = 0, Y_1(q - 1)\}},$$

which enters the outer SISO. The latter works as in standard serially concatenated codes.

C. Metric computation

A keypoint for a successful decoding process is metric computation, according to (18). Hereafter, the metrics are computed in the hypothesis of absence of multicast and inter-symbol interference.

By substituting back (8) and (4) in (17) and with the above hypothesis, we can write, with reference to (5):

$$[Y_1(q - 1), Y_1(q)] = \sqrt{\frac{p_1}{t}} M_1^H H_1 X_1(q - 1) [I_t, \tilde{W}_1(q)] + M_1^H [Z_1(q - 1), Z_1(q)],$$

where $Z_1(q) = [\tilde{z}(qt), \ldots, \tilde{z}(qt + t - 1)]$. By remembering that both $X_1(q - 1)$ and $\tilde{W}_1(q)$ are unitary matrices, the resulting metric can be written:

$$\text{LLR}_j^q(q) = \frac{1}{2} \text{tr} \left\{ (W_j - W_0) Y_1^H(q) \Phi_1 Y_1(q - 1) \right\},$$

where

$$\Phi_1 = \left[ M_1^H M_1 + 2 \frac{p_1}{t} M_1^H \tilde{C}_1 A_1 \tilde{A}_1^H \tilde{C}_1^H M_1 \right]^{-1} - [M_1^H M_1]^{-1},$$

is a $t \times t$ matrix and $A_1 = \text{diag}(a_1,0,\ldots,a_{1,L-1}) \otimes I_r$.

Now, we prove the following proposition:

Proposition 1: The proposed estimator, together with differential space-time encoding, is insensitive to matrix $\Theta$ in (16), and, thus, performs the same as the true MMSE estimator.

Proof: First, notice that, thanks to (16), we have:

$$\hat{s}_1(n) = \Theta^H \tilde{s}_1^{(\text{MMSE})}(n),$$

and thus, keeping into account that the channel coherence time spans at least two space-time codewords:

$$[Y_1(q - 1), Y_1(q)] = \Theta^H \tilde{Y}_1^{(\text{MMSE})}(q - 1), Y_1^{(\text{MMSE})}(q) .$$

Also, substituting (16) into (23), we obtain:

$$\Phi_1 = \Theta^{-1} \tilde{\Phi}_1^{(\text{MMSE})} (\Theta^H)^{-1} .$$

Substituting (25) and (26) into (22), we obtain:

$$\text{LLR}_j^q(q) = \text{LLR}_j^{(\text{MMSE})}(q).$$

V. Simulation results

Fig. 5 shows the performance of the receiver with the blind estimator proposed in this paper, for different values of the system parameters. Each curve in the figure is marked with a triple of numbers, identifying respectively the number of
For example, condition 1) is the same for all users. There is no need to replicate it many times for a multiuser receiver. Also, a part of the algorithm is common to all users, so there is no need to repeat it many times for a multiuser receiver. For example, condition 1) is the same for all users.

VI. CONCLUSIONS

In this paper, we have proposed a new receiver for multi-antenna systems in presence of multipath fading and multiuser interference. This new receiver projects the received signal in the subspace generated by the MMSE matrix filter, without any other information but the spreading sequence of the user of interest.

Although a single-user receiver was dealt with in this paper, it is straightforward to generalize it to a multiuser receiver. Also, a part of the algorithm is common to all users, so there is no need to replicate it many times for a multiuser receiver. For example, condition 1) is the same for all users.

We have seen that the blind estimator in all cases shown has the same performance as the MMSE filter. This is a result of differential encoding, as it has been proved in the paper.

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