Opening angle of loose-snow avalanches

1. Introduction

Loose-snow avalanches are well-diffused phenomena in mountain areas. Despite their frequency of occurrence, as the few literature available confirm, in the past, the researchers have not drawn attention to the phenomenon. On the contrary, the interest is held by slab avalanches that are more dangerous, large and responsible of most of the damages and fatalities (McClung and Schaerer, 2006; Maggioni and others, 2013; Barbero and others, 2013a). However, although loose-snow avalanches are smaller in size and interest smaller volumes of snow, they may be triggered in any period of the year: frequent are the cases in which alpinists are shot by discharges in high altitude ice faces or gullies in summer.

Loose-snow avalanches, or point avalanches, can form both in dry and wet snowpacks and are easy to recognise because of their shape (Schweizer and others, 2003). Differently from slab avalanches, where a fracture propagates in the snowpack (Chiaia and others, 2008) and large masses are involved in the initial movement, in loose-snow avalanches the initial volume is small, e.g. less than $10^4$ m$^3$. A localised lack of cohesion in a small volume of snowpack implies that snow grains fold up on the underneath layers and start to move. The avalanche is triggered naturally when the slope angle exceeds the static-friction angle (or repose angle), named $\phi$. Besides, the avalanche can be triggered by skiers even in thin layers of weak snow (Trautman and others, 2006).

Wet loose-snow avalanches are more frequent in late winter and in spring, dry loose-snow avalanches occur in newly fallen snow or in old surface snow that has faceted (Haegeli and others, 2010). Loose-snow avalanches can frequently take place during the snowy event: an intense snow precipitation with low density snowflakes presupposes low cohesion of the fresh snow and, therefore, small loose avalanches (discharges) happen if, as said, snow depth is larger than a critical value (Haefeli, 1967). Otherwise, a reduction of cohesion due to snowpack properties variation may trigger the avalanche. This is the case of metamorphosed snowpack in which there is a localised increase of snow temperature or water content. Often, the initiation is asso-
associated both to snowpack depth reduction around rock outcrops and to higher snow temperature (McClung and Schaerer, 2006).

The repose angle at which loose-snow avalanches trigger depends upon grain type and geometry. Dendritic crystals have high static-friction angle (up to 80°) and, as much as the grain turns into a round form, the angle decreases to 35°. Moreover, the presence of water may further reduce the value of the angle. Slush snow may avalanche off of slopes of 15° or less (Roch, 1965; McClung and Schaerer, 2006). However, independently from snow type and causes, loose-snow avalanches are characterised by inverted V shape (triangle) with small opening angle, as the ones shown in Figure 1. The apex of the triangle is at the initiation point.

In the past, Haefeli (1967) studied the triggering conditions for loose-snow avalanches. He found that equilibrium conditions in the snowpack are broken if snow depth exceeds the critical value

$$h_0 = \frac{2c}{\gamma} \tan \left( \frac{\pi}{4} + \phi \right)$$

where $c$ and $\gamma$ are snow cohesion and specific weight, respectively. His computations on stresses in the snowpack were made by means of graphical solutions based on the theory of Mohr’s circles, a graphical way to solve stresses problems. It is interesting to notice that other authors used this approach in snow science because of its simplicity (Jaccard, 1966). Obviously, this approach is still used in geotechnics (Parry, 2004), where it was conceived.

One of the key aspects of the approach is the possibility to easily identify if a stress state is admissible for the material, i.e. no rupture occurs. A stress state is not admissible, i.e. the material breaks, if the corresponding Mohr’s circle is secant to the rupture envelope. The rupture envelope is defined empirically by means of laboratory tests and can be approximated by equations. In snow, the relationship between normal and shear stresses, $\sigma$ and $\tau$ respectively, has been found to be accurately described by Mohr-Coulomb failure criterion, which yield line has the form

$$\tau = c + \mu \sigma$$

where $\mu$ is the friction coefficient (Platzer and others, 2007), i.e. $\mu=\tan \phi$. This criterion is extensively applied in soil mechanics, for granular materials in static (Lancellotta, 2008) and dynamic conditions (Nedderman, 2005).

Besides that, the movement of granular materials plays an important role in many natural phenomena (Aranson and Tsimring, 2001), e.g. highly fractured rock masses movements; snow and ice avalanches are phenomena that can be studied in this framework (Savage and Hutter, 1991). Avalanche studies in layers of granular materials showed two types of behaviour, depending upon the thickness of the layer. Daerr and Dou-
ady (1999) showed that a perturbation on a thin layer of glass balls produces an avalanche propagating downhill and laterally, causing triangular tracks, similar to loose-snow avalanches. On the contrary, a perturbation on a thick layer produces an avalanche which propagates both downwards and upwards, with grains located uphill progressively tumbling down because of support. They assessed that the lateral propagation is due only to a small number of grains, which drive into motion some of their lateral neighbours. This chain process is repeated and the edges of the track are formed. Bouchaud and Cates (1998) proposed a simple interpretation of these observations based on an extension of a phenomenological model for surface flows.

Complex natural phenomena can be studied through simplified mechanical models that point on its features and bypass the physical details (De Biagi and Barbero, 2013). In this paper, a possible interpretation of loose-snow avalanches opening angle is given. The model, as shown, is based on a simple mechanical idea, which presupposes that a movement may affect not only the elements situated downhill, but also those situated laterally. The effects are transmitted with a friction mechanism. In order to explain clearly the results, a preliminary section on Mohr’s circles construction and sign conventions is proposed. Then, the model is explained and commented. At the end, a parametric analysis is carried and results are discussed.

2. Summary on Mohr’s circles

In this part, a brief summary on Mohr’s graphical representation of stress states is proposed. These concepts are important for understanding the model described above.

Mohr (1900) first recognised the possibility to represent graphically a stress state on a special reference system \( \sigma_{0\sigma} \), called stress space. His method provides a convenient graphical method for determining the normal and shearing stress on any plane through a point in a stressed body. The conventions used in what follows presuppose that (a) compressive stresses are positive and tensile stresses are negative, and (b) shear stresses are considered positive if they give a clockwise moment about a point above the stressed plane, otherwise negative (Murthy, 2002).

The normal stresses are taken as abscissae and the shear stresses as ordinates. For a given stress state represented by the stress tensor

\[
[\sigma] = \begin{bmatrix}
\sigma_x & \tau_{xy} \\
\tau_{xy} & \sigma_y
\end{bmatrix}
\]  

it is possible to define and to plot on the stress space two points, \( P_1 \) and \( P_2 \) (see Figure 2), whose coordinates are

\[
P_1 = (\sigma_x; \tau_{xy}) \\
P_2 = (\sigma_y; -\tau_{xy})
\]

Stress state \( P_1 \) acts on the vertical plane, while stress state \( P_2 \) acts on the horizontal plane.

The mid-point of segment \( P_1 P_2 \), named \( C \), stays on the \( \sigma \)-axis at abscissa \( (\sigma_x + \sigma_y)/2 \), see Figure 2. The circle passing through the two points and centred in \( C \) has radius, \( r \), equal to

\[
r = \frac{1}{2} \sqrt{(\sigma_y - \sigma_x)^2 - 4\tau_{xy}^2}. 
\]

The two intersections between the circle and \( \sigma \)-axis have abscissae

\[
\begin{align*}
\frac{\sigma_x + \sigma_y}{2} - \frac{1}{2} \sqrt{\sigma_y - \sigma_x} &= \frac{1}{2} \sqrt{\sigma_y - \sigma_x} - \frac{1}{2} \tau_{xy}^2 \\
\frac{\sigma_x + \sigma_y}{2} + \frac{1}{2} \sqrt{\sigma_y - \sigma_x} &= \frac{1}{2} \sqrt{\sigma_y - \sigma_x} + \frac{1}{2} \tau_{xy}^2
\end{align*}
\]

which are the eigenvalues of matrix \( [\sigma] \), i.e. the principal stresses \( \sigma_1 \) and \( \sigma_3 \). The graphical solution is mainly used for finding the orientation of the principal planes if the normal and shear stresses on the surface of the prismatic element are known. The procedure considers a special point, called origin of planes, represented by point \( P_0 \) in Figure 2, which is the intersection between a vertical line conducted from point \( P_1 \), and a horizontal line conducted from \( P_2 \). The lines joining point \( P_0 \) with the intersections of the circle with \( \sigma \)-axis are the directions associated with the principal stresses.

Moreover, it is possible to find, given a rupture envelope in the stress space, the value of the rupture angle of the specimen. Considering the reference system made by the directions, which correspond to the principal stresses, the rupture angle is the angle formed by the line joining the points.
3. Stresses in a uniform snowpack

The distribution of stresses in natural snow slabs has been studied in the past (Jaccard, 1966; Haefeli, 1967). The slab central region, where slope parallel gradients are approximately zero and where stress metamorphosis has a stabilising effect, is called the neutral zone (Perla, 1975). The snowpack is modelled as an infinitely extended layer inclined to the horizontal at an angle of $\theta$. Locally, the snowpack is considered uniform in thickness, $H$, and material properties, elastic modulus, $E$, and Poisson’s ratio, $\nu$. A coordinate system is set on snow surface, see Figure 3.

The equilibrium equations, which consider the six components of the stress tensor, can be expressed as

\[
\begin{cases}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + F_x = 0 \\
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + F_y = 0 \\
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial y} + F_z = 0
\end{cases}
\] (7)

where $F_i$ are the components of the volume force along the axes,

\[
\begin{align*}
F_x &= -g \rho \sin \theta \\
F_y &= 0 \\
F_z &= -g \rho \cos \theta
\end{align*}
\] (8)

g is the acceleration of gravity (9.81 m/s$^2$) and $\rho$ is the local density of the snowpack. In the hypotheses of infinite slope, the partial derivatives with respect of $x$ and $y$ are null. The equilibrium equations simplify in

\[
\begin{align*}
\frac{\partial \tau_{xz}}{\partial z} &= g \rho \sin \theta \\
\frac{\partial \tau_{yz}}{\partial z} &= 0 \\
\frac{\partial \sigma_z}{\partial z} &= g \rho \cos \theta
\end{align*}
\] (9)

The dilatations along $x$ and $y$ axes, $\varepsilon_x$ and $\varepsilon_y$, respectively, are null. Supposing the material linear elastic and isotropic,

\[
\begin{align*}
\varepsilon_x &= 0 : \quad \sigma_x - \nu (\sigma_y + \sigma_z) = 0 \\
\varepsilon_z &= 0 : \quad \sigma_z - \nu (\sigma_y + \sigma_z) = 0
\end{align*}
\] (10)

In the same manner, the distortions on $xy$ and $yz$ planes, $\gamma_{xy}$ and $\gamma_{yz}$, respectively, are null. Within the same hypotheses,

\[
\begin{align*}
\gamma_{xy} &= 0 : \quad \tau_{xy} = 0 \\
\gamma_{yz} &= 0 : \quad \tau_{yz} = 0
\end{align*}
\] (11)

The integration of the previous eqs. (9), (10) and (11), leads to the following solution:

\[
\begin{align*}
\sigma_x &= \frac{\nu}{1-\nu} \sigma_z \\
\sigma_y &= \frac{\nu}{1-\nu} \sigma_z \\
\sigma_z &= \cos \alpha \int_{-h}^0 g \rho \, dz \\
\tau_{xy} &= 0 \\
\tau_{xz} &= \sin \alpha \int_{-h}^0 g \rho \, dz \\
\tau_{yz} &= 0
\end{align*}
\] (12)

where $h$ is the depth of the point in which the stresses are computed ($0 < h < H$). In case of purely elastic case, $\nu$ is Poisson’s ratio (Perla, 1975). In case of purely viscous case, $\nu$ is the viscous analogue of Poisson’s ratio (Perla, 1973). The term $\nu / (1-\nu)$ can be related to creep angle, and consequently, to the density (Haefeli, 1963).

Substituting $h=0$ in the previous eqs. (12), the stress state on snow surface is computed:

\[
\begin{align*}
\sigma_x &= \sigma_y = \sigma_z = 0 \\
\tau_{xy} = \tau_{xz} = \tau_{yz} &= 0
\end{align*}
\] (13)

4. A discrete model for damage propagation in a loose snowpack

In this section, a novel model for triggering propagation, focusing on the opening angle in loose snow avalanches, is described and analysed. The discretisation
of snowpack surface through simple shapes and mutual contact, leads to a propagation model, which, in some ways, may relate to domino effect, i.e. a cumulative effect is produced when one event sets off a chain of similar events (Stronge, 2004).

Let us consider the uppermost layer constituting the snowpack as a series of small elements (of infinitesimal dimensions) laying on the underneath layers (which are not considered for the moment). A sketch of the model is represented in Figure 4. Each element in the model is able to transmit its action to its neighbour elements with normal pressure (along the columns) or shear (along the rows).

Supposing that the avalanche triggers in element A of Figure 4, since the internal cohesion reduces, the self weight of the particle cannot be transmitted to the underneath layers. Self-weight of element A is turned into a normal pressure on element B and into a shear stress on elements $C_1$ and $C_2$. As a matter of evidence, we suppose that the point avalanche spreads laterally if the actions are larger than the corresponding internal resistances.

Since the purpose of the note does not relate to loose snow avalanche triggering, no requirements are need for the initial mechanism. In this sense, one may suppose that the elements interact in a complex manner and the failure might be due to evolving critical stages in the neighbouring elements, as proposed by Bak and others (1988). As stated, the effort of this simple research is devoted to the analysis of the opening angle.

The propagation mechanism remains in the $xy$ initial plane. The stress tensor on the surface of the snowpack is null, see the previous section. This state can be represented on $\sigma\tau$-stresses plane with a point centred in the origin (i.e. a circle with radius equal to zero). In case of normal pressure (as the case of element B), the original Mohr state turns into a circle passing per the origin and point $N_1$, representing $\sigma_z$ value, as represented in left-hand side of Figure 5. On the contrary, in case of shear (as the case of elements $C_1$ and $C_2$), Mohr state is a circle centred in the origin with radius $\tau_{xy}$, as represented in right-hand side of Figure 5. The new stress states can be admissible or not, depending if the circle is internal or secant to the yield bounds. Plain circles of Figure 5 refer to admissible stress states, dotted circles to non-admissible stress states.

For the analysis of the lateral spread of the effects of the triggering of a point avalanche, it is necessary to work on the evolution of the stress state of elements $C_1$ and $C_2$. At the initial stage, as already stated, a point centred in the origin represents the stress state. Since the tangential stress $\tau_{xy}$ increases, the stress state can be expressed by a circle centred in the origin with radius $\tau_{xy}$. As much as the value of $\tau_{xy}$ increases, i.e. the radius of the circle increases, the circle is more and more close to the rupture envelope. For a given value $\tau_{xy}$, Mohr’s circle is tangent to the rupture envelope in point $R$, see Figure 6.

The stress state at this precise point is now analysed in the way explained in paragraph 2.3. The force condition at the moment of rupture is shown in Figure 6. For the sign conventions, $\sigma_{xy}$ referred to the face orthogonal to $x$-axis is negative, while $\sigma_{xy}$ referred to the face orthogonal to $y$-axis is positive. The initial points for tracing the circle are

$$P_1 = (0; -\tau_{xy}^*)$$
$$P_2 = (0; \tau_{xy}^*).$$

For the analysis of the lateral spread of the effects of the triggering of a point avalanche, it is necessary to work on the evolution of the stress state of elements $C_1$ and $C_2$. At the initial stage, as already stated, a point centred in the origin represents the stress state. Since the tangential stress $\tau_{xy}$ increases, the stress state can be expressed by a circle centred in the origin with radius $\tau_{xy}$. As much as the value of $\tau_{xy}$ increases, i.e. the radius of

### Figure 4

A sketch of the discrete domino model. The top element starts to move and its action is transmitted via direct pressure or friction to the neighbour elements generating a chain reaction.

Rappresenatione del modello discreto. L’elemento sommiate inizia a muoversi e la sua azione è trasmessa agli elementi limitrofi tramite un’azione diretta o uno sfregamento. Si realizza quindi un’azione a catena.
the stresses in the principal reference system, the points identifying Mohr's circle are
\begin{align*}
Q_1 &= (\sigma_1; 0) \\
Q_2 &= (\sigma_3; 0).
\end{align*}

Therefore, it is possible to find the new origin of planes, \(O_Q\), with the same procedure previously described.

It is now possible to repeat the same reasoning in the principal reference system. The origin of planes, \(O_Q\), is found to be coincident with point \(Q_2\). The tangency point between Mohr’s circle and the rupture envelope, as previously stated, is named with letter \(R\), see Figure 6. The line \(Q_0R\) represents the rupture plane, with inclination \(\alpha\) in reference to the plane on which the greater principal stress acts, i.e. \(\sigma_1\).

Being the relationship between friction coefficient and friction angle
\begin{equation}
\mu = \tan \phi
\end{equation}
the rupture angle is now investigated. The angle \(\phi\) represents the inclination of the rupture envelope with respect to \(\sigma\)-axis.

Consider the triangle \(\Delta MRO\) of Figure 7: segment \(MR\) is perpendicular to segment \(RO\), because of the tangency between the rupture envelope and Mohr’s circle.
For a geometry theorem on circles and triangles, which states that an angle inscribed in a circle is half of the central angle that subtends the same arc on the circle,

$$\angle ROQ_1 = 2 \angle RQ_0 Q_1 = 2\alpha. \quad (17)$$

Angles $\angle ROQ_1$ and $\angle ROQ_0$ are supplementary, therefore

$$\angle ROQ_0 = \pi - 2\alpha. \quad (18)$$

Since the sum of the internal angles of a triangle is equal to a straight angle ($\pi$), the following expression can be applied to triangle $\Delta MRO$,

$$\phi + \frac{\pi}{2} + \pi - 2\alpha = \pi. \quad (19)$$

Thus, the relationship between the rupture angle, $\alpha$, and the friction angle, $\phi$, can be written

$$\alpha = \frac{\phi + \pi}{2}. \quad (20)$$

Considering that the plane on which the principal stress, $\sigma_1$, acts is rotated $45^\circ$ anti-clockwise, the angle between $x$-axis and the rupture plane is

$$\delta = \alpha - \frac{\pi}{4} = \frac{\phi}{2}. \quad (21)$$

as can be seen in Figure 8.

As found, the rupture plane is inclined of an angle $\delta$ with respect to $x$-axis, i.e. the direction parallel to avalanche track. The same phenomenon may be related to element $C_1$, which is symmetric to the one considered above with respect to the vertical direction. At the apex of the avalanche, i.e. the triggering point, the two rupture planes form an angle of $2\delta$, which is the opening angle. As a first outcome of the previous analysis, the opening angle is affected only by the value of the friction angle, not by the cohesion of the material. This aspect is important, as much as the fact that slope geometry does not enter in the expression of the opening angle. This fact may explain the observed fact that geometrically similar loose-snow avalanches occur in the same area, independently from slope characteristics.

5. Conclusions

Since no systematic studies have been conducted on loose-snow avalanches, few considerations on the capabilities of the model are made. Its effectiveness is, thus, based on simple observations and on the values assumed by the friction angle, $\phi$, as found in literature (Roch, 1965; McClung and Schaerer, 2006).

As a geometric construction, $\delta$-angle and half of the opening angle, $\psi/2$, are alternate interior angles, and, because $x$-axis coincides with the vector gradient of snow surface, the two angles are equal. Therefore, the opening angle of the loose-snow avalanche is

$$\psi = 2\delta = \phi. \quad (22)$$

Following this result, the greater the friction angle, the greater the opening angle.

Classifying the snow for its water content, the following distinction can be made. In dry disaggregated snow, the friction angle depends on the shape of the
grains. Angle $\phi$ is about 35° in case of rounded forms and can increase up to 80° for dendritic crystals. On the contrary, in wet snow, the value of the friction angle is influenced by water content: $\phi$ reduces as much as water content approaches to saturation.

Independently from that, the common friction angle that can characterise the snow of point avalanches ranges from 20° to 40°. This fact implies that the computed opening angle ranges, as well, between 20° and 40° (Roch, 1965; McClung and Schaerer, 2006).

Although this result is preliminary, the model seems to centre the observed, but never recorded and classified, values. Despite the presence of a limited literature on the topic, the evaluation of the opening angle has important implications in the risk analysis. In particular, it can be useful for the computation of the area affected by the avalanche and, consequently, for locating the elements at risk. In fact, it gives indications about the possibility of interaction between a loose-snow avalanche and constructions, such as masts, electric poles or buildings, and infrastructures. The spatial variability can be taken into account in decision tools on road management (Zischg and others, 2005).

As shown, the opening angle strictly depends on the friction angle of the snow. In the future, experimental studies would be conducted directly on the snowpack in order to get accurate measures of this parameter and its correlation with snow density, slope exposure, and meteorological conditions (Barbero and others, 2013b). Moreover, in order to support this study, a detailed and extended survey on real loose-snow avalanches has to be planned. In this case, opening angles and local topography can be estimated with aerial photogrammetrical photos and surveys onsite.

References


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