Original Citation:

Availability:
This version is available at: http://porto.polito.it/2599556/ since: November 2016

Publisher:
ELSEVIER

Published version:
DOI:10.1016/j.proeng.2015.02.015

Terms of use:
This article is made available under terms and conditions applicable to Open Access Policy Article ("Creative Commons: Attribution-Noncommercial-No Derivative Works 3.0") , as described at http://porto.polito.it/terms_and_conditions.html

Porto, the institutional repository of the Politecnico di Torino, is provided by the University Library and the IT-Services. The aim is to enable open access to all the world. Please share with us how this access benefits you. Your story matters.

(Article begins on next page)
Method for calculating the application factor $K_A$ in gears subjected to variable working conditions

Francesca Curà

*Department of Mechanical and Aerospace Engineering, Politecnico di Torino, corso Duca degli Abruzzi 24, Torino, Italy*

**Abstract**

The present paper aims to propose a method, in ISO Standard environment, in order to calculate the $K_A$ application factor when the nominal torque is not defined. A procedure is presented, based on the Miner damage rule, in order to process a given load spectrum and to calculate a value of the equivalent tangential force including variable amplitude effects, useful for bending and pitting life calculation. A practical case is presented, based on a recorded vehicle mission, referring to the input gear of a transfer box for industrial truck. A comparison is also given with the ISO Standard results.

**1. Introduction**

Several methods can be found in literature in order to predict the load capacity of spur and helical gear drives. The conventional design procedure compares the gear teeth strength with a calculated stress value, in both bending and pitting cases. The most common approach used in calculation of these stress values, as widely described in ISO Standard 6336 [1-5] and in AGMA Standard 2001-D04 [6], involves the use of influence factors, derived from results of research and field service [7].

Influence factors may be distinguished between two categories, factors which are determined by gear geometry or which have been established by convention and factors which account for several influences and which are dealt with as independent of each other; in this last group is included the application factor $K_A$. The factor $K_A$ adjusts the nominal load $F_t$ in order to compensate for incremental gear loads from external sources. These additional loads are largely dependent on the characteristics of the driving and driven machines, as well as the masses and stiffness of...
the system, including shafts and couplings used in service.

For applications such as marine gears and others subjected to cyclic peak torque (torsional vibrations) and designed for infinite life, the application factor can be simply defined as the ratio between the peak cyclic torques and the nominal rated torque. The nominal rated torque is defined by the rated power and speed. It is the torque used in the load capacity calculations.

If the gear is subjected to a limited number of known loads in excess of the amount of the peak cyclic torques, this influence may be covered directly by means of cumulative fatigue or by means of an increased application factor, representing the influence of the load spectrum.

ISO Standard Part 6 [5] proposes a procedure for the calculation of an equivalent load in order to obtain the application factor value $K_A$; this factor, defined as the ratio between the equivalent torque and the nominal one, may give a first estimation of the service life already in the design phase.

The present paper aims to propose a method, in ISO Standard environment, in order to calculate the $K_A$ factor when the nominal torque is not defined, but above all when the gear has been already designed at least from the cinematic point of view and specific variable amplitude loading data are given.

In particular, in this work a procedure is presented based on the Miner damage rule, as in ISO Standard, in order to process a given load spectrum and to calculate a value of the equivalent tangential force that includes the variable amplitude effects; this force value is useful for bending and pitting calculation of the service life. This procedure is applied to the input gear of a transfer box for industrial truck. Variable amplitude loads refer to a vehicle mission recorded during different routes (highway, mixed route, mountain road, ..): both measured torque and cycles number are available to be processed.

Obtained results in terms of $K_A$ factor have been compared to those calculated by means of the classical ISO Standard procedure.

2. State of the art: ISO Standard procedure for calculating the application factor $K_A$ under variable loads

ISO Standard part 6 [5] proposes a calculation method of service life of gears under variable load [8]. In particular, in addition to the classical tables (usually available for components) where an application factor $K_A$ is indicated for working characteristics of both driving and driven machines, it describes a method for a design case where the Woehler-damage line is simplified by ignoring all damage which occurs at stresses below some limit stress corresponding to the fatigue limit of a material for pitting and bending strength.

The application factor $K_A$ for a given load spectrum, in terms of transmitted torque, is simply defined as the ratio between the equivalent torque $T_{eq}$ and the nominal torque $T_n$ as:

$$K_A = \frac{T_{eq}}{T_n}$$  \hspace{1cm} (1)

As already quoted, the application factor $K_A$ has to be determined for tooth root brakeage (bending) and for pitting resistance; the equivalent torque is defined by the damage Miner rule where the slope $p$ of the Woehler-damage line and the number of load cycles $N_{l,ref}$ at the reference point (endurance limit cycles) have to be known.

By resuming, the procedure described in [5] needs the material Woehler-damage curves (slope $p$ and $N_{l,ref}$) and an hypothesis of nominal torque.

The equivalent torque $T_{eq}$ is defined by the following equation:

$$T_{eq} = \left(\frac{n_1T_1^p + n_2T_2^p + ... + n_iT_i^p}{n_1 + n_2 + ... + n_i}\right)^{\frac{1}{p}}$$  \hspace{1cm} (2)

where $i=1, 2, ...$ is the number of bin, $n_i$ is the number of cycles for bin $i$, $T_i$ is the torque for bin $i$.

The focus point in the procedure is that the number of bins to be used in equation (2) cannot be predetermined.
A torque $T_i$ in the bin $i$ can be replaced by a torque $T_j$ in a new bin ($j=i+1$), so that the damage caused by the torque $T_i$ is the same as that caused by the torque $T_j$ expressed by the following equation:

$$T_i^p n_i = T_j^p n_j$$ (3)

The load bins have to be denoted as $(T_i, n_i)$ and numbered in descending order of torque, where $T_1$ is the highest torque. Then the cycles $n_j$ at torque $T_j$ are equivalent in terms of damage to a larger number of cycles $n_{ja}$ at lower torque $T_2$, where:

$$n_{ja} = n_j \left( \frac{T_1}{T_2} \right)^p$$ (4)

If $n_{2e} = n_2 + n_{ja}$, then the bins 1 and 2 can be replaced by a single bin $(T_2, n_{2e})$ and so on.

This procedure has to be stopped when $n_{ie}$ reaches the endurance limit cycles $N_{Lref}$.

3. Method for calculating the application factor $K_A$ of a component under variable loads

The method proposed in the present paper for determining the application factor $K_A$ in the case of variable amplitude loads aims to be a completion of the ISO Standard [5] procedure when the gear has been already designed at least from the cinematic point of view.

The heart of the method involves the use of a fatigue curve of the component instead of the Woehler-damage curve of the material.

So, being the pitch diameters of the engaging gears known, it is possible to calculate the tangential forces (involved in ISO Standard basis equations) corresponding to each torque level.

The way of thinking that is the basis of this method allows also the specific calculation of the service life of gears, even if details about this procedure are here omitted for sake of brevity.

It may also be utilised when the nominal torque value is unknown, as an example when experimental data are available about loading conditions of the transmission.

As ISO Standard [5], this method utilises the Miner damage rule and the corresponding exponent $p$ of Woehler-damage curves (slope $p$) for both bending and pitting cases.

The substantial difference is that the number of bins to be used in the Miner damage rule refers to the loading blocks that really are damaging the gears. In other words, being the amount of damage depending on the stress level, it is necessary to calculate the tangential force level $F_{td}$ for which the damage entity can be considered as zero.

This force limit $F_{td}$, belonging to the fatigue curve of the gear (slope $p$), simply corresponds to the endurance limit cycles $N_{Lref}$.

The procedure will be described in detail in the following, referring only to the bending case for sake of brevity. Analogous relationships may be obtained for pitting.

According to [1], [3], the tooth root stress $\sigma_F$ is the maximum tensile stress at the surface in the root and it may be calculated by the following equation:

$$\sigma_F = \sigma_{FO} K_A K_y K_{Fa} K_F \beta$$ (5)

where $K_A$ and $K_y$ are respectively application and dynamic factors, $K_F \beta$ and $K_{Fa}$ face and transverse load factors, $\sigma_{FO}$ is the nominal tooth stress expressed by:

$$\sigma_{FO} = \frac{F_L}{b m_n} Y_F Y_S Y_\beta Y_B Y_{DT}$$ (6)
where $F_t$ is the nominal tangential load, $b$ is the facewidth, $m_n$ is the normal module, $Y_F$, $Y_S$, $Y_{\beta}$, $Y_b$, $Y_{DF}$ are respectively form, stress correction, helix angle, rim thickness and deep tooth factors.

Equation (5) may be expressed in a compact form as:

$$\sigma_F = \frac{F_t}{b m_n} K A$$  \hspace{1cm} (7)

where:

$$A = K_v K_{F_d} K_{F_l} Y_F Y_S Y_{\beta} Y_B Y_{DF}$$ \hspace{1cm} (8)

The permissible bending stress $\sigma_{FP}$, following [3] (method B), is given by:

$$\sigma_{FP} = \sigma_{F_{\text{lim}}} Y_{NT} \frac{Y_{ST} Y_{\delta_{T}} Y_{R_{relT}} Y_X}{S_{F_{\text{min}}}}$$ \hspace{1cm} (9)

where $\sigma_{F_{\text{lim}}}$ is the nominal stress number for bending from reference test gears, $Y_{ST}$ is the stress correction factor, $Y_{NT}$ is the life factor for tooth root stress expressed as a function of the the number of load cycles $N_L$, $S_{F_{\text{min}}}$ is the minimum required safety factor for tooth root stress, $Y_{\delta_{relT}}$, $Y_{R_{relT}}$, $Y_X$ are respectively relative notch sensitivity, relative surface and size factors.

Also equation (9) may be expressed in a compact form as:

$$\sigma_{FP} = \sigma_{F_{\text{lim}}} Y_{NT} B$$ \hspace{1cm} (10)

where:

$$B = \frac{Y_{ST} Y_{\delta_{relT}} Y_{R_{relT}} Y_X}{S_{F_{\text{min}}}}$$ \hspace{1cm} (11)

This procedure makes the tooth root stress corresponding to the permissible bending stress as:

$$\sigma_F = \sigma_{FP}$$ \hspace{1cm} (12)

So, by substituting equations (7) and (10) respectively in equation (12), the following relationship may be obtained:

$$\frac{F_t}{b m_n} K A = \sigma_{F_{\text{lim}}} B Y_{NT}$$ \hspace{1cm} (13)

Equation (13) provides the basis of the procedure, provided that the number of endurance limit cycles $N_{L_{\text{ref}}}$ is known. For the case of bending stress, $N_{L_{\text{ref}}}$ generally corresponds [1-5] to $3 \times 10^6$, so:

$$\frac{F_t}{b m_n} K A = \sigma_{F_{\text{lim}}} B \left[ \frac{3 \times 10^6}{N_L} \right]^{\text{exp}}$$ \hspace{1cm} (14)
where \( exp \) means the general term for slope \( p \) of the material fatigue curve. If \( N_L \) coincides with the number of endurance limit cycles \( N_{L,ref} = 3 \times 10^6 \), also the tangential force \( F_t \) coincides with the load level for which the damage entity can be considered as zero, so \( F_t = F_{tD} \); in this way, the \( F_{tD} \) force value can be easily obtained:

\[
F_{tD} = \frac{\sigma_{F,lim} bm_n B}{A} \tag{15}
\]

Once \( F_{tD} \) is known for that gear, the Miner procedure may be run, by considering only the fatigue cycles that really are damaging the gear. Rearranging equation (14) in order to calculate the application factor \( K_A \), it gives:

\[
F_t K_A N_L^{exp} = \frac{bm_n}{A} \sigma_{F,lim} B \left[3 \times 10^6\right]^{exp} \tag{16}
\]

or, in a more compact form:

\[
\left[ F_t K_A \right]^{1 \exp} N_L = C_F \tag{17}
\]

where:

\[
C_F = \left[ \frac{bm_n}{A} \sigma_{F,lim} B \right]^{1 \exp} \left(3 \times 10^6 \right) \tag{18}
\]

Then, the equivalent tangential force \( F_{eq} \) can be obtained by the following equation:

\[
F_{eq} = \left[ \frac{\sum_{i=1}^{L} n_{iw} F_{ti}^{exp}}{\sum_{i=1}^{L} n_{iw}} \right]^{exp} \tag{19}
\]

where only the tangential force levels \( F_{ti} \) (respectively running for \( n_{iw} \) cycles) higher than \( F_{tD} \) are taken into account. The numbers of cycles \( n_{iw} \) means both a number of cycles (as in equation (2)) or, depending on the available data, an already weighted (subscript \( w \)) number of cycles, as described in the following paragraph related to a practical case.

Finally, the application factor \( K_A \) in the case of variable amplitude loads can be obtained:

\[
K_A = \frac{F_{eq}}{F_{tD}} \tag{20}
\]

4. Practical case: input gear of a transfer box for industrial truck

The practical case developed in the present work is the application factor \( K_A \) calculation of the input gear (gear 1, number of teeth \( z_f = 36 \)) (see Figure 1) of the transfer box for industrial truck in on road conditions. Material has been chosen as hardening steel.
Experimental data are available on this gear in terms of torque level and cycles number (root number of the transfer box input shaft). In other words, for a 100 km standard distance for each route (Borgo Taro – Pontremoli, Torino – Aosta, … and so on), a mission of the vehicle has been run and the corresponding reference parameters have been measured in different driving conditions (see Table 1), following an established percent division (7% Motorway, 70% Mixed Route and Mountain, 23% Urban route).

It can be noted than positive values for torque refer respectively to engine driving condition, while negative values to engine braking conditions.

Data of Table 1 have been therefore processed to obtain a weighted mission (see Table 2), for which only global values of cycles number are available to make easier the following procedure.

![Fig. 1. Gear 1 geometry.](image)

### 5. Results and discussion

Table 3 reports the coefficients values related to gear 1 obtained following the ISO Standard formula (method B) [1]. The dynamic factor has been calculated referring to the maximum speed of the vehicle (90 km/h). The exponent of the Woehler-damage line indicated as \( p \) in paragraph 2 corresponds to the general exponent indicated as \( \text{exp} \) in paragraph 3.

Coefficients have been obtained following the indications of the manufacturer about material, surface finish, and so on.

Table 4 reports steps and corresponding calculations for obtaining the application factor \( K_A \) of gear 1, following the ISO Standard procedure described in paragraph 3, on the basis of the vehicle experimental data. In particular, the first column shows the bin number \( (i=1, 2, \ldots) \), columns two and three respectively the corresponding torque \( T_i \) and tangential force \( F_{ti} \) values for gear 1, the fourth column shows the weighted number of cycles \( n_{iw} \) (being the procedure applied to a mission), the fifth one reports the equivalent cycles from row above \( n_{iaw} \), the fifth one the total number of cycles \( n_{iew} \) and the last one shows the switch \( s_w \). As it can be observed, bin 6 (shadowed row) corresponds to splitting limited and infinite service life (switch \( s_w \) passes from 0 to 1).

<table>
<thead>
<tr>
<th>Teeth number</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal module [mm]</td>
<td>6</td>
</tr>
<tr>
<td>Normal pressure angle</td>
<td>22.5°</td>
</tr>
<tr>
<td>Reference hole angle</td>
<td>18°</td>
</tr>
<tr>
<td>Reference pitch diameter [mm]</td>
<td>227.116</td>
</tr>
</tbody>
</table>

Table 1. Vehicle mission.

<table>
<thead>
<tr>
<th>Roots number of the transfer box input shaft</th>
<th>7% Motorway</th>
<th>70% Mixed Route and Mountain</th>
<th>23% Urban route</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_i ) [Nm]</td>
<td>Borgo Taro</td>
<td>Torino</td>
<td>Piacenza</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
Referring to weighted data of Table 2, it can be noted that in this case only positive torque values have been taken into account, being the most damaging for gear 1. As it can be observed from data of Table 4, force value stands between bins 7 and 8, so only four bins (8-11) have been excluded from the calculation of equation (20) also the application factor $K_{a}$.

Then, following the procedure object of this work, $F_{eq}$ force value ($F_{eq} = 88946$ N) has been calculated by means of equation (15). Referring to weighted data of Table 2, it can be noted that in this case only positive torque values have been taken into account, being the most damaging for gear 1. As it can be observed from data of Table 4, $F_{eq}$ force value stands between bins 7 and 8, so only four bins (8-11) have been excluded from the calculation of $F_{eq}$ and from $K_{a}$. From equation (19) the equivalent force has been obtained ($F_{eq} = 99553$ N) and finally from equation (20) also the application factor $K_{a}$ of gear 1: $K_{a} = 1.119$.
By varying other parameters, as an example the gear facewidth, it is possible to tune as one likes the application factor $K_A$.

Table 4. Loads data and ISO Standard sequence for $K_A$ calculation.

<table>
<thead>
<tr>
<th>i</th>
<th>$T_i$ [Nm]</th>
<th>$F_{ti}$ [N]</th>
<th>$n_{iw}$</th>
<th>$n_{nor}$</th>
<th>$n_{aw}$</th>
<th>$s_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14147</td>
<td>213552</td>
<td>27</td>
<td>64.8</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12799</td>
<td>193204</td>
<td>518</td>
<td>1540</td>
<td>582.8</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>11452</td>
<td>172871</td>
<td>2989</td>
<td>13516</td>
<td>4529</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>10105</td>
<td>152537</td>
<td>50825</td>
<td>2E+05</td>
<td>64341</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>8757</td>
<td>132189</td>
<td>2E+05</td>
<td>2E+06</td>
<td>4E+05</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>7410</td>
<td>111856</td>
<td>6E+05</td>
<td>1E+07</td>
<td>2E+06</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>6063</td>
<td>91522</td>
<td>1E+07</td>
<td>2E+08</td>
<td>3E+07</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>4716</td>
<td>71189</td>
<td>1E+07</td>
<td>5E+09</td>
<td>3E+08</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>3368</td>
<td>50841</td>
<td>2E+07</td>
<td>4E+11</td>
<td>5E+09</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>2021</td>
<td>30507</td>
<td>1E+07</td>
<td>6E+15</td>
<td>4E+11</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>674</td>
<td>10174</td>
<td>3E+07</td>
<td></td>
<td>6E+15</td>
<td>1</td>
</tr>
</tbody>
</table>

6. Conclusions

The method described in the present paper aims to complete the ISO Standard calculation procedure for as concerns the application factor $K_A$ in the case of variable amplitude loads.

In particular, a case of industrial vehicle subjected to real working conditions has been presented.

The substantial difference between ISO Standard and the present work is the use respectively of a Woehler-damage line of the material and a specific fatigue curve of the component.

As a consequence of that, ISO Standard method may be used already in the design phase, while the present method needs the knowledge of almost the cinematic data of the gear.

Nevertheless, being the calculation specific for a gear, the real amount of damage may be obtained for both application factor $K_A$ and service life determination.

Results show that ISO Standard procedure provides an application factor value a little more conservative than that specifically obtained.

It may be finally observed that the method presented in this paper allows to monitor the variation of the application factor $K_A$ on the basis of the given values for gear coefficients, as an example for facewidth.

Acknowledgements

The author expresses her gratitude to Ing. Lino Coppo of FPT Industrial S.p.A. for experimental data and helpful suggestion.

References