[Article] Hausdorff-Besicovitch Dimension of Lake Nasser Coastline from Satellite Imagery

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Hausdorff-Besicovitch Dimension of Lake Nasser Coastline from Satellite Imagery

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Abstract
The paper is proposing a method for evaluating the Hausdorff-Besicovitch dimension of natural coastlines from satellite images.

A fractal dimension is a statistical index which is describing the complexity of a given pattern embedded in given spatial dimensions. In particular, this index is providing a measure of the capacity the considered fractal pattern has to fill the space in which it is embedded [1-3].

Fractals became popular with the works of Benoit Mandelbrot, starting from his 1967 paper where he discussed the fractional dimensions [4] (for references on the fractional approach to calculus, see [5]). In [4], Mandelbrot illustrated the fractal dimensions by citing a previous work by Lewis Fry Richardson, who was discussing how a coastline's measured length could change with the length of the rigid stick used for measurements. In this manner, the fractal dimension of a coastline was linked to the number of rigid sticks, required to measure the coastline, and to the scale of the used stick [6].

Actually, using the Fry Richardson example cited by Mandelbrot, we can also imagine an “experimental method” for the investigation of the fractal dimension of natural environments, such as the abovementioned coastlines of seas and lakes, and many others. For instance, in [7], where we proposed a link between fractal dimensions and entropies, we have considered the rim of the Grand Canyon. Before discussing the experimental method, which we will illustrate in this paper by means of a part of the coastline of Lake Nasser, a definition of fractal dimension is necessary. Let us note that several formal mathematical definitions exist: here we use that of the Hausdorff-Besicovitch dimension.

Felix Hausdorff proposed a fractional dimension in 1918 as a measure of the surface roughness [8]. Since several methods to calculate this dimension had been developed by Abram Samoilovitch Besicovitch, today the Hausdorff dimension is also known as the Hausdorff-Besicovitch dimension. It is given by (in the following formula, N stands for the number of sticks used to cover the coastline and h for the scaling factor): $D = \log (N(h)) / \log (1/h)$, in the limit as h goes to zero. The experimental approach to this fractal dimension is similar to that proposed in [10].

For the evaluation of the fractal dimension of the natural coastline of Lake Nasser, the vast water reservoir in southern Egypt and northern Sudan, let us use the Figure 1 (the image is coming from Google Earth imagery; in a previous article [11], we considered this lake for giving a recurrence plot of its level from altimetric data).

Figure 1: A part of Lake Nasser in Google Earth imagery.

Using GIMP, the GNU Image Manipulation Program, and in particular its Pixelize filter (https://docs.gimp.org/2.6/en/plug-in-pixelize.html), we can render the image in the Figure 1 as a series of images composed by large color blocks. Figure 2 shows them in binary images, made of black blocks in a white background. Therefore, by means of GIMP, we have a series of binary images suitable for the experimental evaluation of the fractal dimension of the coastline. Our "sticks" are the sizes of the blocks in the Pixelize filtered images.

Figure 2: Using the Pixelize filter of GIMP we have a series of binary images that we use for evaluating the fractal dimension of the coastline.

By means of an image segmentation, for instance that proposed for some previous experiments [19-21], we can easily determine the number N of sticks we have to use to measure the length of the consider coastline of the lake, in each of the binary images of the Figure 2. In the Figure 3, we can see the corresponding Pixelize filtered coastlines. Of them, we can easily determine the lengths as N(h), for each considered scale h.

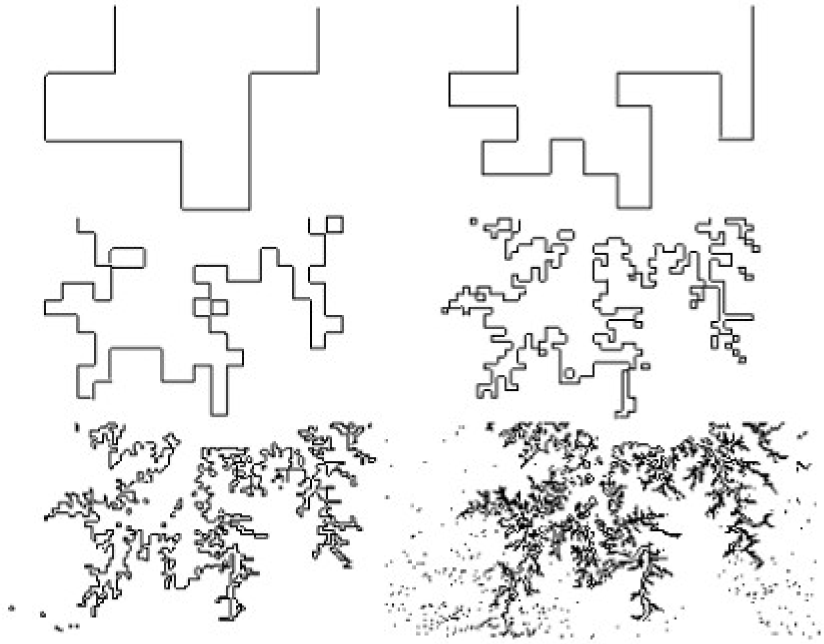

Figure 3: The Pixelize filtered coastlines considered for the evaluation of the fractal dimension, obtained by an image segmentation [20].

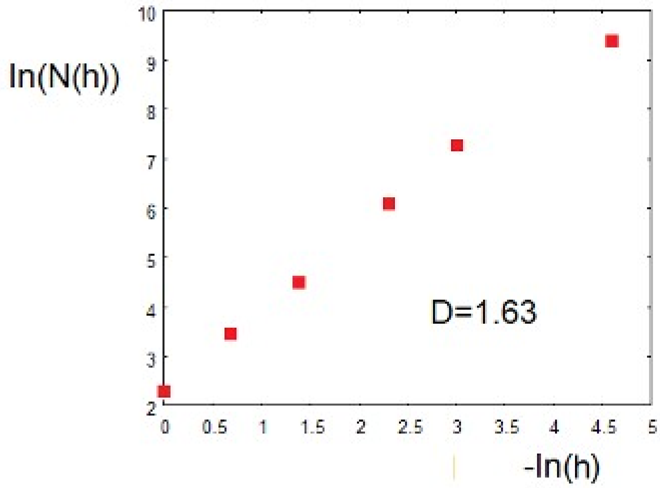

Figure 4: Data obtained from the image segmentation and used for evaluating the fractal dimension of the coastline in the Figure 1.

It is easy to plot log (N(h)), log (1/h) as in the Figure 4 (natural logarithms). The slope of the best-fit line of the data in the Figure 4 approximates the Hausdorff-Besicovitch dimension [10]. In our example, the fractal dimension D of this part of the coastline is about 1.63.

It seems that this same approach can be applied to determine the fractal dimensions of other natural objects. However, further studies are necessary. Some examples are under investigation.

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